

Nonlocal nondegenerate optical parametric amplifier based on genuine multipartite entanglement

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We propose a scheme for realizing nonlocal nondegenerate optical parametric amplifier by means of genuine four-mode entanglement. The nondegenerate optical parametric amplifier is regarded as a unitary transformation from two input fields to two outputs. Two inputs and two outputs are located at the remote stations, respectively. Employing off-line four-mode entanglement, homodyne detection, classical communication, and local displacement, the nonlocal nondegenerate optical parametric amplifier can be achieved. This scheme can be implemented with a setup that is experimentally accessible at present.

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The nondegenerate optical parametric amplifier (NOPA) can be described with the following input-output unitary transformation:

$$\hat{a}_{out}^s = \sqrt{G}\hat{a}_{in}^s + \sqrt{G-1}\hat{a}_{in}^{i\dagger}, \quad (1)$$

$$\hat{a}_{out}^i = \sqrt{G-1}\hat{a}_{in}^{s\dagger} + \sqrt{G}\hat{a}_{in}^i, \quad (2)$$

where $\hat{a}_{in}^{s(i)}$ and $\hat{a}_{out}^{s(i)}$ represent the annihilation bosonic operators of the input and output signal (idler) fields, respectively, and G is the power gain. Usually NOPA is closely related to phase-insensitive amplification, which only involves Eq. (1). The quantum state \hat{a}_{in}^s is amplified by an amount \sqrt{G} , and at the same time an additive noise $\sqrt{G-1}\hat{a}_{in}^{i\dagger}$ must be attached to maintain commutation relation ($[\hat{a}_{out}^s, \hat{a}_{out}^{s\dagger}] = 1$) of the amplifier output. The ultimate limits imposed by quantum mechanics were addressed many years ago [1–3]. This intrinsic noise, intimately linked with measurement theory and the no-cloning theorem, gives rise to many inextricable restrictions on the manipulations of quantum states. Despite these limitations, the phase-insensitive amplifier is application rich, in particular, in optical communication. Furthermore, NOPA is of particular importance for quantum information and communication, for example, the generation of two-mode Gaussian entangled state [Einstein-Podolsky-Rosen (EPR) entangled state] [4,5], and quantum cloning [6–8].

Numerous apparatuses can be used for accomplishing the phase-insensitive amplification, for instance, solid-state laser amplifiers [9], and the schemes based on the linear optics and feedforward [10]. However, the idler output in these schemes is not always accessible, e.g., in a laser amplifier this mode is scattered into vibrational modes of atoms. Although NOPA can be achieved directly by type-II parametric interaction [4,5], there is a lot of technic trouble in the implementation process for coupling efficiently, mediated by a nonlinearity, the input signal, and idler fields into the pump field. We define this type of NOPA as the local NOPA since the two input and two output fields are at the same location. The localization limits its application in quantum information and

communication. A recent trend in quantum information science is that the nonlinear interaction are efficiently replaced by off-line entanglement, classical communication, and local operation [11,12]. The protocol of substituting the in-line parametric amplifier with linear optics, feedforward, and an off-line EPR source has been suggested [10,13,14], such that the trouble to couple the input signal and idler fields with the pump field in a nonlinear crystal can be overcome. In Ref. [13] local NOPA and quantum nondemolition (QND) interaction based on off-line squeezers, linear optics, and measurements is proposed. Recently, implementing nonlocal interaction between spatially distant quantum network nodes becomes very interesting work. Nonlocal QND interaction can be implemented [15] by using an off-line four-mode cluster state [16], which is the genuine multipartite entanglement. In this paper we show that NOPA can be achieved nonlocally by using an off-line four-mode entanglement, homodyne detection, classical communication, and local displacement to replace the in-line nonlinear interaction. In the proposed system, all two inputs and two outputs locate four remote stations, respectively. The used off-line four-mode entanglement is a class of genuine multipartite entangled states, which is different from Greenberger-Horne-Zeilinger (GHZ) and cluster states and can be generated by combining two beams from two different pairs of EPR entangled beams on a beam splitter.

First we must prepare a four-mode entangled state and distribute them to four remote stations. As shown in Fig. 1, two pairs $(\hat{a}_{EPR1}, \hat{a}_{EPR2})$ and $(\hat{b}_{EPR1}, \hat{b}_{EPR2})$ of EPR entangled states are utilized to generate the genuine four-mode entangled state. The EPR entangled beams have the very strong correlation property, such as both their sum-amplitude quadrature variance $\langle \delta(\hat{X}_{a(b)_{EPR1}} + \hat{X}_{a(b)_{EPR2}})^2 \rangle = 2e^{-2r_1(2)}$ and their difference-phase quadrature variance $\langle \delta(\hat{Y}_{a(b)_{EPR1}} - \hat{Y}_{a(b)_{EPR2}})^2 \rangle = 2e^{-2r_1(2)}$ are less than the quantum noise limit, where r_1 and r_2 are the squeezing factor, and \hat{X} and \hat{P} describe the amplitude and phase quadrature of the optical field with the canonical commutation relation $[\hat{X}, \hat{P}] = 2i$. One of the EPR entangled beams (\hat{a}_{EPR2}) is combined with one of the other EPR pair (\hat{b}_{EPR2}) on a beam splitter BS_0 with reflectivity rate R and transmission rate $T = 1 - R$. The output fields are expressed by

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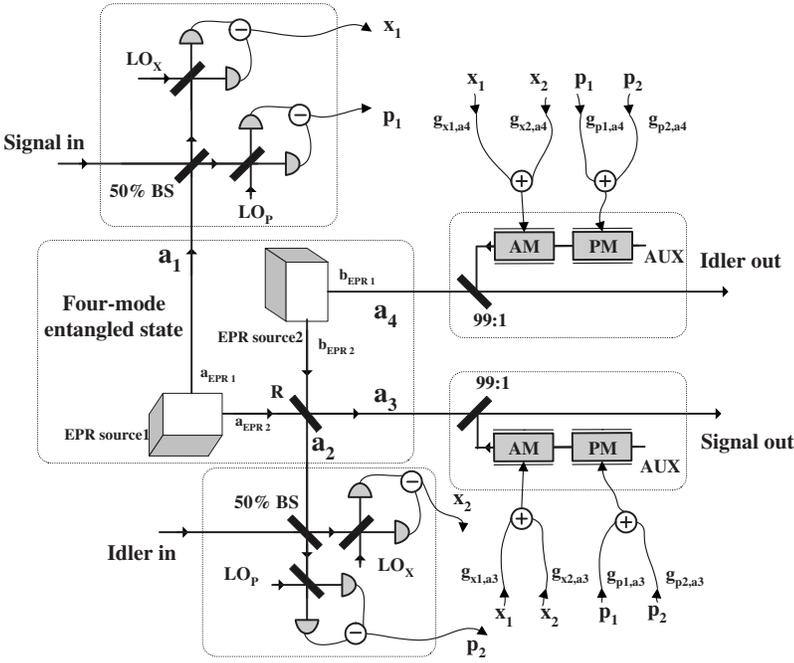


FIG. 1. Schematic diagram of nonlocal NOPA with genuine four-mode entanglement. BS, beam splitter; LO, local oscillator; AM, amplitude modulator; PM, phase modulator; AUX, auxiliary beam.

$$\hat{c}_i = \sqrt{1-R}\hat{b}_{EPR2} - \sqrt{R}\hat{a}_{EPR2},$$

$$\hat{c}_r = \sqrt{R}\hat{b}_{EPR2} + \sqrt{1-R}\hat{a}_{EPR2}. \quad (3)$$

So, the four-mode entangled state for nonlocal NOPA is described with the annihilation operator $\hat{a}_1 = \hat{a}_{EPR1}$, $\hat{a}_2 = \hat{c}_i$, $\hat{a}_3 = \hat{c}_r$, $\hat{a}_4 = \hat{b}_{EPR1}$. The properties of the four-mode entangled state for nonlocal NOPA will be discussed in the following by its quantum correlation variances of the amplitude and phase quadratures (position and momentum). The quantum correlations of the four-mode entangled state for nonlocal NOPA can be obtained easily with $\langle \Delta^2(X_{a_1} - \sqrt{R}X_{a_2} + \sqrt{1-R}X_{a_3}) \rangle < \mathcal{L}_{SN}$, $\langle \Delta^2(\sqrt{1-R}X_{a_2} + \sqrt{R}X_{a_3} + X_{a_4}) \rangle < \mathcal{L}_{SN}$, and $\langle \Delta^2(P_{a_1} + \sqrt{R}P_{a_2} - \sqrt{1-R}P_{a_3}) \rangle < \mathcal{L}_{SN}$, $\langle \Delta^2(\sqrt{1-R}P_{a_2} + \sqrt{R}P_{a_3} - P_{a_4}) \rangle < \mathcal{L}_{SN}$, where \mathcal{L}_{SN} is the shot noise limit. It clearly shows that this state exhibits the full inseparability (genuine four-party entanglement) according to the criteria of detecting genuine multipartite continuous variables (CV) entanglement [17]. In each of the above inequalities, the position and momentum correlation variance involves three-party combination from the four entangled modes. Now we compare CV four-mode cluster state [16] and GHZ state with the four-mode entangled state for nonlocal NOPA. The inseparability of CV four-mode cluster state are expressed by the correlations of position and momentum of $\langle \Delta^2(X_1^C + X_2^C + X_3^C) \rangle < \mathcal{L}_{SN}$, $\langle \Delta^2(X_3^C + X_4^C) \rangle < \mathcal{L}_{SN}$, and $\langle \Delta^2(P_1^C - P_2^C) \rangle < \mathcal{L}_{SN}$, $\langle \Delta^2(P_2^C - P_3^C + P_4^C) \rangle < \mathcal{L}_{SN}$ [16,18], and CV four-partite GHZ state by total position $\langle \Delta^2(X_1 + X_2 + X_3 + X_4) \rangle < \mathcal{L}_{SN}$ and relative momentum $\langle \Delta^2(P_i - P_j) \rangle < \mathcal{L}_{SN}$ ($i, j = 1, 2, 3, 4$) [16–18]. We can see that some two-party correlation of position or momentum components are included in the criterion inequalities of CV four-partite cluster state and GHZ state.

For implementing the nonlocal NOPA the four submodes of the four-mode entangled state $\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4$ are distributed

into the remote signal input, idler input, signal output, and idler output stations, respectively. The mode \hat{a}_1 (\hat{a}_2) and the input signal \hat{a}_{in}^s (idler \hat{a}_{in}^i) are combined at a 50/50 beam splitter inside the signal (idler) input station. Then the two input stations independently perform the homodyne measurements on themselves two output beams from their 50/50 beam splitter to extract the amplitude and phase quadratures of optical fields. The obtained quadratures (x_1, p_1) , (x_2, p_2) are respectively

$$x_1 = \frac{1}{\sqrt{2}}(\hat{X}_{a_1} + \hat{X}_{a_{in}^s}), \quad p_1 = \frac{1}{\sqrt{2}}(\hat{P}_{a_1} - \hat{P}_{a_{in}^s}),$$

$$x_2 = \frac{1}{\sqrt{2}}(\hat{X}_{a_2} - \hat{X}_{a_{in}^i}), \quad p_2 = \frac{1}{\sqrt{2}}(\hat{P}_{a_2} + \hat{P}_{a_{in}^i}). \quad (4)$$

The input stations send these measured results to both remote output stations, respectively. After receiving these measurement results, each output station displaces the mode by means of a 1/99 beam splitter with an auxiliary beam, the amplitude and phase of which have been modulated via the amplitude (AM) and phase (PM) modulators by the sum signals (x_{a_3}, p_{a_3}) , (x_{a_4}, p_{a_4}) , respectively. The position $(x_{a_3}$ and momentum values $p_{a_3})$ of the modulated sum signals for displacing the entangled mode \hat{a}_3 is equal to

$$x_{a_3} = g_{x1,a_3}x_1 + g_{x2,a_3}x_2 = \sqrt{\frac{1}{1-R}}\hat{X}_{c_{in}^s} + \sqrt{\frac{R}{1-R}}\hat{X}_{c_{in}^i}$$

$$- \sqrt{R}\hat{X}_{b_{EPR2}} + \sqrt{\frac{1}{1-R}}\hat{X}_{a_{EPR1}} + \frac{R}{\sqrt{1-R}}\hat{X}_{a_{EPR2}},$$

$$p_{a_3} = g_{p1,a_3} p_1 + g_{p2,a_3} p_2 = \sqrt{\frac{1}{1-R}} \hat{P}_{c_{in}^s} - \sqrt{\frac{R}{1-R}} \hat{P}_{c_{in}^i} - \sqrt{R} \hat{P}_{b_{EPR2}} - \sqrt{\frac{1}{1-R}} \hat{P}_{a_{EPR1}} + \frac{R}{\sqrt{1-R}} \hat{P}_{a_{EPR2}}, \quad (5)$$

where $g_{x1,a_3} = -g_{p1,a_3} = \sqrt{2/(1-R)}$ and $g_{x2,a_3} = g_{p2,a_3} = -\sqrt{2R/(1-R)}$ are scaling factors, and that for displacing the entangled beam \hat{a}_4 is equal to

$$x_{a_4} = g_{x1,a_4} x_1 + g_{x2,a_4} x_2 = -\sqrt{\frac{R}{1-R}} \hat{X}_{c_{in}^s} - \sqrt{\frac{1}{1-R}} \hat{X}_{c_{in}^i} + \hat{X}_{b_{EPR2}} - \sqrt{\frac{R}{1-R}} (\hat{X}_{a_{EPR1}} + \hat{X}_{a_{EPR2}}),$$

$$p_{a_4} = g_{p1,a_4} p_1 + g_{p2,a_4} p_2 = \sqrt{\frac{R}{1-R}} \hat{P}_{c_{in}^s} - \sqrt{\frac{1}{1-R}} \hat{P}_{c_{in}^i} - \hat{P}_{b_{EPR2}} - \sqrt{\frac{R}{1-R}} (\hat{P}_{a_{EPR1}} - \hat{P}_{a_{EPR2}}), \quad (6)$$

where $g_{x1,a_4} = g_{p1,a_4} = -\sqrt{2R/(1-R)}$ and $g_{x2,a_4} = -g_{p2,a_4} = \sqrt{2/(1-R)}$. Corresponding to the transformation $\hat{A} \rightarrow \hat{D}^+ \hat{A} \hat{D} = \hat{A} + (\hat{X}_m + i\hat{P}_m)/2$ in the Heisenberg representation, the displaced fields at the remote output stations can be expressed as

$$\hat{a}_{out,a_3}^s = \sqrt{\frac{1}{1-R}} \hat{a}_{in}^s + \sqrt{\frac{R}{1-R}} \hat{a}_{in}^{i\dagger} + \sqrt{\frac{1}{1-R}} (\hat{a}_{EPR1}^\dagger + \hat{a}_{EPR2}),$$

$$\hat{a}_{out,a_4}^i = \sqrt{\frac{R}{1-R}} \hat{a}_{in}^{s\dagger} + \sqrt{\frac{1}{1-R}} \hat{a}_{in}^i - (\hat{b}_{EPR2}^\dagger + \hat{b}_{EPR1}) + \sqrt{\frac{R}{1-R}} (\hat{a}_{EPR1} + \hat{a}_{EPR2}^\dagger). \quad (7)$$

Thus we obtain the output signal and idler fields of NOPA simultaneously [same as Eqs. (1) and (2)] at the remote stations with gain factor $G=1/(1-R)$ in the case of perfect quantum correlations ($r_1, r_2 \rightarrow \infty$). The gain of NOPA can be adjusted by the reflectivity rate R of the beam splitter BS_0 . Under the realistic condition with the imperfect four-mode entanglement, the extra noises are added in the output signals of NOPA.

In conclusion, we have proposed a scheme for a nonlocal optical nondegenerate parametric amplifier using a genuine four-mode entanglement. This genuine four-mode entanglement is a class of multipartite entangled states, which can be generated with two pairs of EPR entangled beams and linear optics. The inseparability criteria of the proposed four-mode entangled state are different from that of the CV four-partite cluster state and GHZ state. The nonlocal NOPA can be implemented by off-line four-mode entanglement, homodyne detection, classical communication, and local displacement. Due to its nonlocality the proposed NOPA might be extensively applied in a CV quantum information network.

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